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13. ABSTRACT (*Maximum 200 words*)

In this research, an innovative tool, called the *Distributed Transfer Function Method* (DTFM), is developed for modeling and analysis of complex flexible military structures. With the developed method, the eigensolutions, dynamic response and stability of a large class of flexible systems subject to a variety of forcing sources, multi-body coupling and constraints are precisely and efficiently predicted. Computational algorithms for modeling and analysis of different structural components such as beams, plates and shells with laminated layers, as well as their assemblies, have been developed. The DTFM is also shown to be useful for exploring innovative techniques in vibration controller design and smart structure design. The research results have important applications in optimal design and vibration control of helicopter rotors with blades, robotized armament systems, rotating machinery, vehicle frame-suspension systems, and tank cannons and machine guns.

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Distributed parameter systems, flexible structures, distributed transfer functions, Green's function, vibration control, structural dynamics, analytical solution methods

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# Modeling and Analysis of Complex Flexible Systems Using Distributed Transfer Functions

Final Progress Report

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February 27, 1997

US ARMY RESEARCH OFFICE

Grant No. DAAH04-93-G-0066

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**Table of Contents**

## A. STATEMENT OF THE PROBLEM STUDIED

This research develops a new analytical and numerical method, namely the Distributed Transfer Function Method (DTFM), for the modeling and analysis of general complex flexible systems that are assemblages of multiple continua and lumped parameter systems. The research introduces a distributed transfer function formulation, based on which, the eigensolutions, static and dynamic response of complex flexible systems subject to a variety of forcing sources, and damping, inertial and gyroscopic effects are thoroughly investigated. The distributed transfer function method is also used to study control-structure interactions which are essential to active/passive structural control and smart structures. The conducted research is fundamental to the dynamics of structures and machines, and has important applications in optimal design of military structures and equipment.

## B. SUMMARY OF RESEARCH RESULTS

The objective of this research is to develop an innovative tool, namely the *Distributed Transfer Function Method* (DTFM), for modeling, analysis and optimal design of complex flexible military structures. The research project has gone through the four stages shown in Fig. 1: fundamental development of DTFM-based synthesis; modeling of flexible components; static and dynamic analysis of complex structural systems, and study on control-structure interaction. In sequel, the results obtained from each stage will be detailed.

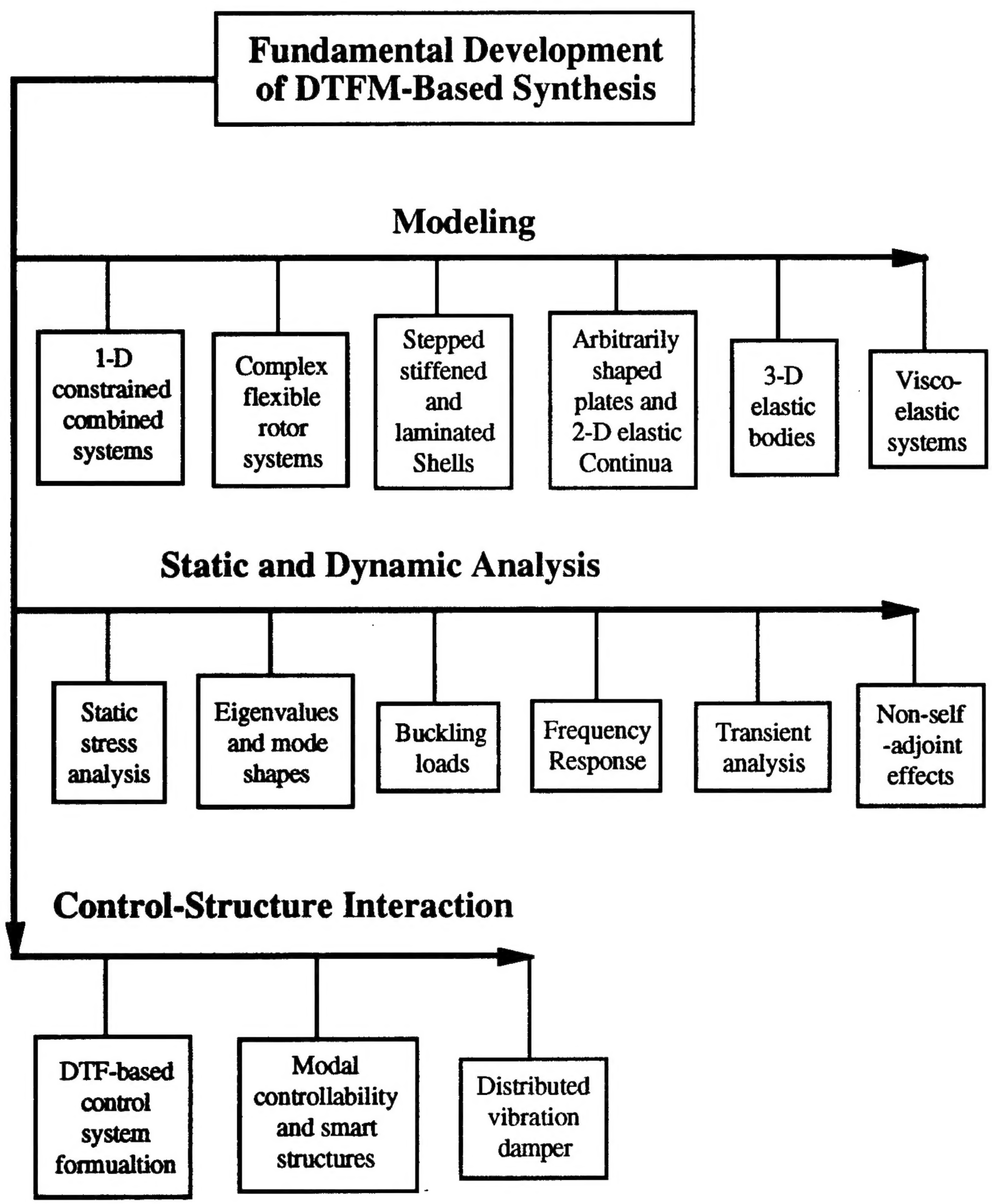



Fig. 1

## 1. Fundamental Development of DTFM-Based Synthesis

The DTFM-based synthesis developed follows four steps as illustrated in Fig. 2. The transfer function synthesis deals with complex flexible systems (CFS) with multiple branches and closed-loops, distributed connection of flexible bodies, and multi-point connection of flexible and lumped subsystems. The synthesis combines the advantages of both the transfer function method and the finite element method, namely the exact and closed form of the distributed transfer function method, and the flexibility of the finite element method in dealing with complex flexible subsystems [J1][1]. Theoretical and numerical work reveals the following important properties of the proposed synthesis:

(a) The synthesis provides exact and closed-form solutions for many CFS, without the need for discretization.

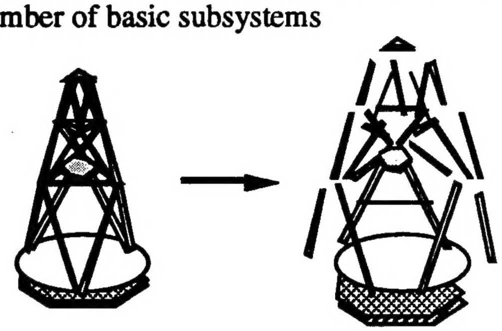
(b) The synthesis does not need to select particular shape functions for specific system parameters and boundary conditions.

(c) The synthesis is efficient in treating various constraints, and multi-body coupling.

(d) The synthesis can precisely predict high frequency responses of CFS.

(e) The synthesis is highly accurate in estimating high-gradient stress distributions of CFS.

Step 1. Decompose the structures into a number of basic subsystems



Step 2. Obtain the analytical/semi-analytical and closed-form transfer functions for each subsystem

Distributed Transfer Functions

$$\eta(x,s)=\int_{\Omega}G(x,\xi,s)\upsilon(\xi,s)d\xi+H(x,s)\gamma(s)$$

Response

External Disturbance

Boundary Disturbance

Step 3. Assemble the subsystems $\Longrightarrow$

Dynamic equilibrium equation

$$K(s)u(s)=q(s)$$

Step 4. Solve for $u(s)$ $\Longrightarrow$

static and dynamic response

mechanical energy and power flow

eigensolutions, stress distributions

. . . . . . . . . . . .

Fig. 2

One unique feature of the distributed transfer function formulation is that different flexible systems are systematically treated with the same algorithm. Inertia, stiffness, axial loads,

[1] [J1] stands for the reference J1 listed in the publication section in pp. 11-12 of this report

gyroscopic and centrifugal forces, circulatory effects, and various boundary and external disturbances are conveniently described without requiring different derivation, which is often the case with other modeling techniques. This feature allows computerized symbolic manipulation for general multi-body systems, and makes the proposed modeling method flexible in dealing with real engineering problems.

Computer programs for implementing the synthesis have developed. These programs are useful in study of the eigenvalue, response, stability and control problems of CFS.

## 2. Modeling of Complex Flexible Systems

*One-dimensional flexible subsystems* [J2]

Determination of the distributed transfer functions of flexible subsystems is essential to the proposed transfer function synthesis. The research first obtains exact closed-form distributed transfer functions of uniform one-dimensional flexible subsystems such as strings, bars, beams, and beam-columns, and then extends the derivation to non-uniform systems such as beams with varying cross-section and mass density. Two numerical methods are developed to estimate the transfer functions of non-uniform systems. Compared to the existing methods the transfer function modeling is more accurate and efficient.

*Constrained/combined flexible systems* [J3, T2][2]

The research investigates flexible systems that are constrained by springs and dampers, and are combined with rigid bodies. This class of systems has wide applications in army equipment and facilities such as robotized armament systems, rotorcraft, missiles and ground vehicles. A systematic modeling and analysis method for constrained/combined flexible systems has been developed. In the method, constraint forces and flexible-rigid body coupling are analyzed using the obtained distributed transfer functions and transfer matrices for the constraints and rigid bodies. Formulas for calculating eigensolutions and frequency response are derived. One advantage of the developed method is that constraints and combined lumped systems are specified in a simple and compact matrix form which is convenient for computer coding and numerical simulation.

As application of the transfer function method, four types of systems have been considered: (a) coupled string-rigid body systems; (b) beams connected with rigid blocks; (c) beam frames and truss systems; and (d) a multi-body vehicle structure. The main concern is on the coupling and interaction of flexible and rigid bodies. Computer programs for calculating natural frequencies and mode shapes, and frequency response are being developed using MATLAB. Numerical analysis and comparison of the proposed method with the existing modeling techniques show many advantages of the transfer function method. It is believed that the computer programs developed in this research will be quite useful for real army applications.

*Complex flexible rotor systems* [J16, T1]

The distributed transfer function synthesis is also applied to rotor systems assembled from multiple flexible and rigid components, and constrained by bearings and gears; see Fig. 3. The method delivers highly accurate and closed-form analytical solutions, and is capable of treating

[2] [T2] stands for the reference T2 listed in the publication section in pp. 11-12 of this report

non-self-adjoint effects of gyroscopic and damping forces, general boundary conditions and multi-body coupling. It is shown that the proposed method provides a useful analysis tool for many problems in rotor dynamics.

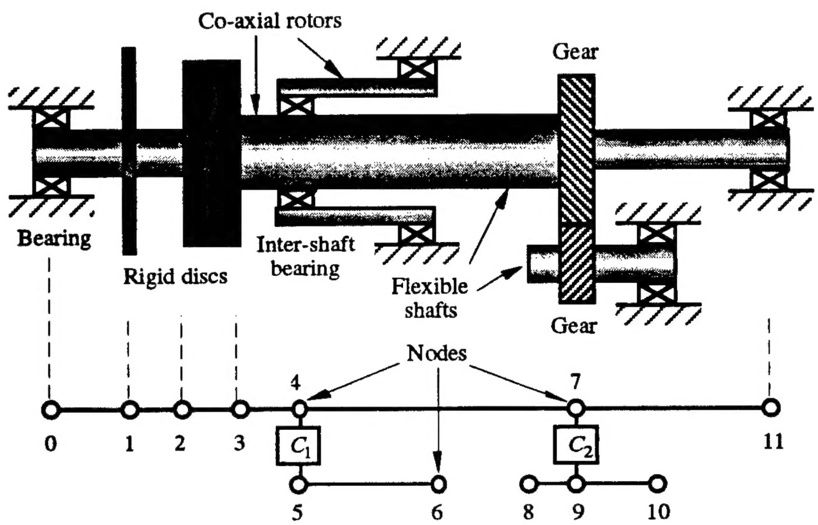



Fig. 3

*Stepped, ring-stiffened and laminated cylindrical shells* [J4, J6, J8]

Stepped and ring-stiffened cylindrical shells, and thick and laminated shells have been modeled analyzed, with interest in tank cannons and other weapon systems. The DTFM is combined with Fourier series expansion to yield highly accurate pre-dictions of shell response, and dis-continuities in the distributions of shell internal forces. An example is shown in Fig. 4 where the internal forces of a cylindrical shell of two stiffeners in free vibration are plotted along the shell longitudinal direction.

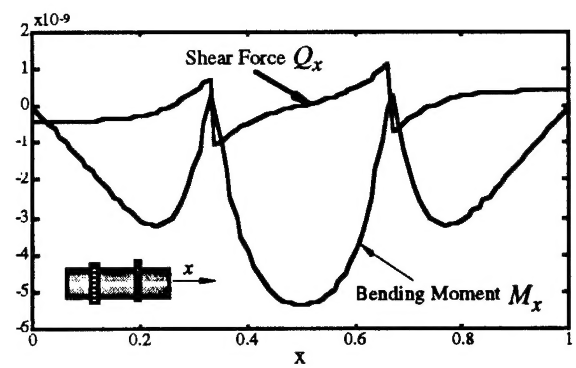



Fig. 4

*Arbitrarily shaped plates and 2-D elastic regions* [J7, J10, J15]

Extending from analysis of one-dimensional continua, the proposed DTFM has found useful and important applications in modeling of complex-shaped plates and 2-D elastic regions. Unlike the cylindrical shells, the complex geometry and boundary conditions of these 2-D continua do not permit Fourier series expansion. To overcome the difficulty, a generalized technique called the Strip Distributed Transfer Function Method (SDTFM) is developed for the two-dimensional continua. In this semi-analytical method, a multi-body region is first decomposed into a number of

basic 2-D subregions (rectangular or sectorial). Each subregion is then divided into a number of strips; the response of each strip is interpolated in the unknown nodal line displacements, which are functions of the strip longitudinal coordinate and time. The nodal line displacements are determined in an exact and closed form by the distributed transfer functions that are defined along the strips. Synthesis of the subregions using the strip distributed transfer functions yields semi-exact and semi-analytical prediction of the static and dynamic response, natural frequencies and buckling loads of the multi-body continuum.

The SDTFM is shown to be much more accurate and efficient than the Finite Element Method (FEM). For instance, consider the calculation of the natural frequencies of a cantilever plate in transverse vibration (Fig. 5a), and the stress distribution of an L-shaped 2-D elastic region (Fig. 5b), on line A-B where the stress $\sigma_y$ becomes singular at point A. Table 1 and Fig. 6 show the high accuracy of SDTFM in predicting higher-mode eigenvalues and high-gradient stresses: with just 4 and 12 strips in the two cases, respectively, SDTFM delivers the same accurate results as obtained by FEM with 900 and 2,700 elements.

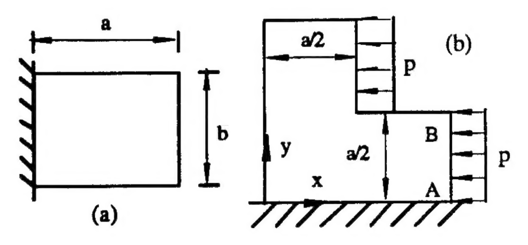



Fig. 5

Table 1. Non-dimensional natural frequencies of a cantilever plate (a/b = 1.5)

| Mode No. | SDTFM 4 strips | FEM 4x4 | FEM 8x8 | FEM 30x30 |
|---|---|---|---|---|
| 1 | 1.5352 | 1.5314 | 1.5346 | 1.5348 |
| 2 | 5.1825 | 5.2300 | 5.1970 | 5.1808 |
| 3 | 9.5422 | 10.252 | 9.7418 | 9.5534 |
| 4 | 17.489 | 18.573 | 17.854 | 17.502 |
| 5 | 23.820 | 25.081 | 24.213 | 23.812 |
| 10 | 58.285 | 64.454 | 60.252 | 58.363 |
| 15 | 87.625 | 94.502 | 91.959 | 88.364 |
| 20 | 124.80 | 157.38 | 137.71 | 125.84 |

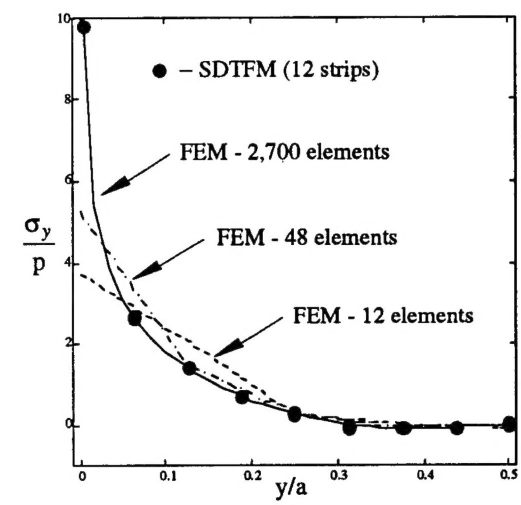



Fig. 6 Stress distribution of L-shaped region on A-B.

The SDTFM is not limited to rectangular and sectorial shapes, it is valid for elastic region with arbitrarily curved boundaries. Through a coordinate transformation, an arbitrarily shaped spatial domain of the continuum is mapped onto a rectangular region, where the isoparametric strip distributed transfer functions of the plate are introduced. Consequently, semi-exact and semi-analytical solutions for various static and dynamic problems can be systematically obtained. As an example, consider the plate in Fig. 7, where the two curved boundaries are described by $a_0(y) = -0.1\sin 3\pi y$ and $a_1(y) = -1.2y(y-1)+0.2$. The calculated nondimensional natural frequencies $\lambda_k$ listed in Table 2

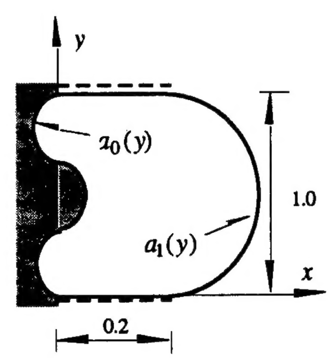



Fig. 7

indicates that only a few strips are needed in SDTFM to obtain the results that require thousands of elements in a finite element analysis. Analysis of other shaped plates has been considered, and also reveals the high accuracy and efficiency of the method.

Table 2. Natural frequencies of the plate with two different curved boundaries (ns - number of strips)

| Method | | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ |
|---|---|---|---|---|---|---|
| SDTFM | ns = 2 | 3.0008 | 5.5244 | 11.3428 | 15.8556 | 16.4639 |
| | ns = 4 | 2.5970 | 4.9815 | 9.1872 | 14.9220 | 14.9308 |
| | ns = 6 | 2.5428 | 4.7922 | 8.6906 | 13.9231 | 14.7647 |
| | ns = 8 | 2.5326 | 4.7326 | 8.5532 | 13.5813 | 14.7411 |
| FEM ($72 \times 50$ elements) | | 2.5407 | 4.8422 | 8.6978 | 13.3843 | 14.8493 |

*3-D elastic bodies* [J17]

The concept of the strip distributed transfer function is further extended to three-dimensional elastic solids. Semi-analytical solutions are obtained using prism distributed transfer functions, which are defined on prisms of the elastic region, instead of strips. Again, one unique feature of the method is to be able to model multi-body continua of arbitrary boundaries, using a smaller number of prisms.

*Visco-elastic systems* [J18]

The DTFM developed is extremely convenient for modeling and analysis of structures made of viscoelastic material, or with viscoelastic damping layers. For a linear viscoelastic continuum (1-D, 2-D or 3-D) its constitutive relation in the Laplace-transform domain can be described by $\bar{\sigma}(x,s) = s\bar{R}(x,s)\bar{\varepsilon}(x,s)$, where $\bar{\sigma}$, $\bar{R}$ and $\bar{\varepsilon}$ are the Laplace transforms of the stress, strain and relaxation function of the continuum, and $s$ is the Laplace transform parameter. By making use of the above $s$-domain constitutive relation, DTFM can directly model and analyze the viscoelastic structure without the need for extra effort. As a result, the proposed method avoids dealing the time-domain convolution integrals that are commonly encountered in many existing analysis technique. Furthermore, through a specific inverse Laplace transform procedure, closed-form transient response of a viscoelastic structure is obtained for the first time.

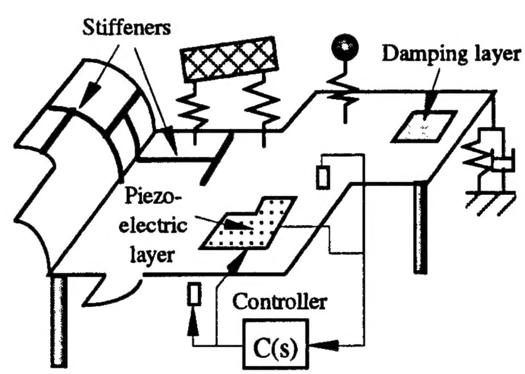



Fig. 8

Besides viscoelastic materials or damping layers, the DTFM synthesis is also capable of modeling structures with piezoelectric layers. Indeed, the method developed indeed provides a useful tool for modeling and analysis of smart structures like the hypothetic one in Fig. 8.

## 3. Static and Dynamic Analysis

As shown in Fig. 1, the SDTFM-based synthesis of a complex flexible system leads to the dynamic equilibrium equation

$$K(s)u(s) = q(s) \tag{1}$$

where $u(s)$ is the vector of the displacements at the points (called nodes) where flexible subsystems are interconnected, $q(s)$ is the vector of effective loads at the nodes, and $K(s)$ is a transcendental dynamic stiffness matrix which is assembled from the distributed transfer functions of all the subsystems. Based on the above equation, analytical or semi-analytical solutions of various static and dynamic problems have been obtained.

*Eigenvalue solutions* [J1, J2, J4, J10, J15]

For the flexible system in free vibration, its characteristic equation is $\det \mathrm{K}(s) = 0$ from which the system eigenvalues $\lambda_k$, $k$ = 1, 2,..., can be determined. Once the eigenvalues are known, the corresponding eigenfunctions are obtained in two steps: first, determine the non-trivial solution $\{\phi_k\}$ of $\mathrm{K}(\lambda_k)\{\phi_k\} = 0$; second, substitute $\{\phi_k\}$ into the transfer function representation of the response of each subsystem to give the mode shape distributions.

In determining the eigenvalues (eigenfrequencies and damping ratios, or buckling loads) of the flexible system, one needs to solve the transcendental characteristic equation $\det \mathrm{K}(s) = 0$. A numerical root searching scheme has been developed based on explicit estimation of the derivative $d\{\det \mathrm{K}(s)\}/ds$. In addition, to assure fast convergency of the scheme, good initial guesses of the roots are made by using the eigenvalue inclusion principles and the spectrum shift approach.

*Static and frequency response* [J1, J7, J8, J10]

In static analysis, the Laplace transform parameter $s$ in Eq. (1) is set zero. For the flexible system under a harmonic excitation of frequency $\omega$, its dynamic response is obtained by setting $s = i\omega$, $i = \sqrt{-1}$, in Eq. (1). Thus, closed-form estimation of static and dynamic displacements and stresses is systematic and efficient, as has been shown in Fig. 6.

*Transient response analysis* [J3, J9, J11, J12, J18]

An innovative method for evaluation of transient response of a complex flexible system with damping and gyroscopic effects, of viscoelastic and smart materials, and under active control actions has been developed. This method obtains closed-form inverse Laplace transform of the s-domain response by using the modal information. Two unique features of the method are: (i) the method does not need orthogonality relations for the system eigensolutions, which actually are difficult to find for the flexible systems in consideration; and (ii) the method does not assume the completeness of the eigenfunctions in a function space.

*Non-self-adjoint effects* [J2, J5, J9, J16, C6, C7][3]

In case study, the research investigated the effects of non-proportional damping, Coriolis acceleration, impact, follower forces, and inertial forces on the dynamic behaviors of complex flexile systems. The stability of the flexible system under those effects are predicted through use

[3] [C6] stands for the reference C6 listed in the publication section in pp. 11-12 of this report

of the closed-form frequency response. The convenience and accuracy of using the distributed transfer functions is obviously observed.

## 4. Control-Structure Interaction

As have shown in the previous sections, the DTFM is accurate and efficient in modeling and analysis of complex flexible systems. The focus of this part of research, therefore, is on the usefulness of the DTFM-based synthesis in design of active/passive controllers for complex flexible systems, and in design of smart structures. Full development of sophisticated control algorithms is beyond the scope of this project. However, to serve as a precursor to future investigations, the following three issues have been selected in the case study.

*4.1. Control system formulation by distributed transfer functions* [J1, J12, C6, T1]

The flexible system in consideration consists of multiple distributed subsystems (e.g., flexible components, damping treatment, and smart material layers) and lumped subsystems (e.g., rigid bodies, oscillators, dampers, and finite-dimensional feedback controllers). Mathematically, the system response is described by a set of coupled partial and ordinary differential equations with complicated boundary and matching conditions. Conventional discretization techniques like FEM usually leads to a model of a large number of degrees of freedom, which usually has computational difficulties in on-line control. In this research, it is shown that for a given frequency spectrum, an accurate model of a few modal coordinates can be easily obtained from the distributed transfer function formulation. With that model, simple and effective controllers can be convenient designed. Furthermore, the study indicates that the DTF-based control system formulation for high-frequency dynamics does not require increased number of unknowns or computation time, which is an attractive feature in control of structural acoustics.

*4.2. Modal controllability and smart structures* [J5, C7]

The response of the controlled structure (see Fig. 9) is governed by:

$$\underbrace{M\ddot{x}+D\dot{x}+Kx}_{\substack{\text{Structural}\\\text{dynamics}}}+\underbrace{\int_0^t R(t-\tau)x(\tau)d\tau}_{\substack{\text{Smart material}\\\text{and constrained}\\\text{damping layers}}}=\underbrace{\int_0^t B_a(t-\tau)}_{\substack{\text{Actuator}\\\text{locations}\\\text{\& dynamics}}}\underbrace{u(\tau)}_{\substack{\text{Control}\\\text{input}}}d\tau$$

A condition on controllability using modal information is obtained as follows:
To control the kth mode,

$$\hat{B}_a(\lambda_k)v_k \neq 0$$

where $\hat{B}_a(s)$ is the Laplace transform of the operator $B_a(t)$, and $\lambda_k$ and $v_k$ are the eigenvalue and corresponding mode shape that are determined by DTFM.

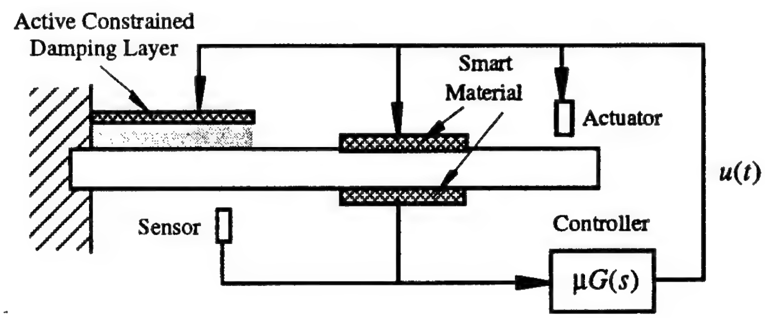



Fig. 9

Uniqueness of the Modal Condition

- **Explicit relationship between controllability and modes to be controlled, and quantitative measure of controllability by $\|v_k\|$**

- **Applicable to structural systems with uncertain/unidentified parameters (e.g., inherent damping, coupling of multiple bodies)**

Utility in Smart Structure Design

- Optimal placement of actuators and active/passive layers, and optimal shape design of smart material and damping layers

- Systematic design of feedback gain $\mu G(s)$ based on the sensitivity coefficient of the close-loop poles $p_k$

$$\frac{dp_k}{d\mu} = \frac{\text{Change in closed - loop pole}}{\text{Change in control gain}} = \left\{\hat{B}_s(\lambda_k)v_k\right\}^T \times G(\lambda_k) \times \hat{B}_s^T(\lambda_k)v_k$$

where the operator $\hat{B}_s(s)$ describes the locations and dynamics of the sensors, and $G(s)$ is a controller transfer function to be designed.

*4.3. Tuned distributed vibration damper*

A novel concept of tuned distributed vibration damper (TDVD) is introduced. A TDVD consists of an elastic continuum with distributed inertia and stiffness, and a boundary conditioner with tunable parameters. The boundary conditioner can be of various forms, including conventional viscous damper, active feedback controller, combination of passive and active components, and smart materials. The main function of the boundary conditioner is to deliver passive/active damping for energy dissipation, while the main function of the bar is to amplify or augment the damping delivered by the boundary conditioner. The preliminary analysis shows that the TDVD can increase the damping in a controlled structure by several orders of magnitude, in a wide frequency region; see the example in Fig. 10.

Continued study of this interesting topic may lead to new ways for intelligent control of structures and equipment in various military applications.

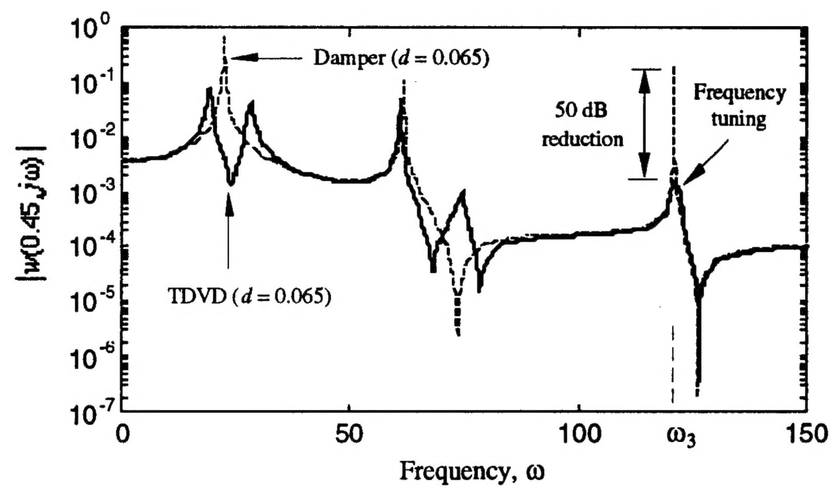



Fig. 10 Frequency responses of a clamped-clamped beam controlled by a TDVD whose boundary conditioner is a damper, and by a pure damper

## C. LIST OF PUBLICATIONS

This project results in 34 publications which are listed as follows

### Refereed Journal Papers

J1. Yang, B., 1994, "Distributed Transfer Function Analysis of Complex Distributed Parameter Systems," *ASME Journal of Applied Mechanics*, Vol. 61. No. 1, pp. 84-92.

J2. Yang, B., and Fang, H., 1994, "A Transfer Function Formulation of Non-Uniformly Distributed Parameter Systems," *ASME Journal of Vibration and Acoustics*, Vol. 116, No. 4, pp. 426-432.

J3. Yang, B., 1995, "Linear Vibration of a Coupled String - Rigid Bar System," *Journal of Sound and Vibration*, Vol. 183, No. 3, June, pp. 383-399.

J4. Zhou, J., and Yang, B., 1995, "A Distributed Transfer Function Method for Analysis of Cylindrical Shells," *AIAA Journal*, Vol. 33, No. 9, September, pp. 1698-1708.

J5. Yang, B., 1995, "Modal Controllability and Observability of General Mechanical Systems," *ASME Journal of Vibration and Acoustics*, Vol. 117, No. 4, October, pp. 510-515.

J6. Yang, B., and Zhou, J., 1995, "Analysis of Ring-Stiffened Cylindrical Shells," *ASME Journal of Applied Mechanics*, Vol. 62, No. 4, December, pp. 1005-1014.

J7. Zhou, J., and Yang, B., 1996, "Strip Distributed Transfer Function Method for the Analysis of Plates," *International Journal of Numerical Methods in Engineering*, Vol. 39, No. 11, June, pp. 1915-1932.

J8. Zhou, J., and Yang, B., 1996, "Three-Dimensional Stress Analysis of Thick Laminated Composite Cylindrical Shells and Panels," *AIAA Journal*, Vol. 34, No. 9, September, pp. 1960-1964.

J9. Yang, B., 1996, "Integral Formulas for Non-Self-Adjoint Distributed Dynamic Systems," *AIAA Journal*, Vol. 34, No. 10, October, pp. 2132-2139.

J10. Yang, B., and Zhou, J., 1996, "Semi-analytical Solution of 2-D Elasticity Problems by the Strip Distributed Transfer Function Method," *International Journal of Solid and Structures*, Vol. 33, No. 27, pp. 3983-4005.

J11. Yang, B., 1996, "Closed-Form Transient Response of Distributed Damped Systems, Part I: Modal Analysis and Green's Function Formula," *ASME Journal of Applied Mechanics*, Vol. 63, No. 4, December, pp. 997-1003.

J12. Yang, B., 1996, "Closed-Form Transient Response of Distributed Damped Systems, Part II: Energy Formulation for Constrained and Combined Systems," *ASME Journal of Applied Mechanics*, Vol. 63, No. 4, December, pp. 1004-1010.

J13. Yang, B., and Zhou, J., 1996, "Strip Distributed Transfer Function Analysis of Circular and Sectorial Plates," *Journal of Sound and Vibration*, in press.

J14. Yang, B., and Wu, X., 1997, "Transient Response of One-Dimensional Distributed Systems: A Closed-Form Eigenfunction Expansion Realization," *Journal of Sound and Vibration*, in review.

J15. Yang B, and Park, D.-H., 1997, "Analysis of Plates with Curved Boundaries Using Isoparametric Strip Distributed Transfer Functions, " *International Journal of Numerical Methods in Engineering*, in review.

J16. Fang, H., and Yang, B., 1997, "Modeling, Synthesis and Dynamic Analysis of Complex Flexible Rotor Systems," *Journal of Sound and Vibration*, in review.

J17. Park, D.-H., and Yang B, 1997, "Prism Distributed Transfer Function Method for 3-D Elastic Solids," in preparation for publication.

J18. Wu, X., and Yang, B., 1997, "Closed-Form Transient Analysis of Distributed Viscoelastic Beams," in preparation for publication.

## Conference Papers

C1. Yang, B., and Fang, H., 1993, "Transfer Function Formulation of Non-Uniformly Distributed Parameter Systems," *Proceedings of the 14th ASME Biennial Conference on Mechanical Vibration and Noise*, Albuquerque, New Mexico, DE-Vol. 61, pp. 79-86.

C2. Yang, B., 1994, "A Transfer Function Formulation for Complex Distributed Parameter Systems," *Proceedings of the International Conference on Vibration Engineering*, Beijing, China, 15-18, pp. 35-40.

C3. Yang, B., 1994, "Linear Vibration of a Coupled String-Rigid Bar System," *Proceedings of the International Conference on Vibration Engineering*, Beijing, China, pp. 91-96.

C4. Yang, B., 1994, "Modal Controllability and Observability of General Mechanical Systems," *Proceedings of the 1994 ASME Winter Annual Meeting*, Chicago, November 6-11, DE-Vol. 75, pp. 363-370.

C5. Zhou, J., and Yang, B., 1995, "Distributed Transfer Function Analysis of Ring-Stiffened Cylindrical Shells," *Proceedings of the 1995 Design Engineering Technical Conferences*, Boston, DE-Vol. 84-3, Part C, pp. 861-868.

C6. X. Wu, and Yang, B., 1996, "Transient Analysis of Controlled Flexible Structures with Viscoelastic Damping Material," *Proceedings of the 1996 SPIE Symposium on Smart Structures and Materials*, San Diego, pp. 26-29.

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C12. Park, D.-H. Park, Yang, B., 1997, "Static and Dynamic Response of Plates with Curved Boundaries," submitted to *the 16th ASME Biennial Conference on Mechanical Vibration and Noise*.

## Book Chapters

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T1 Yang, B., 1995 "Wave Motion in a Flexible Rod with Tuned Boundary Impedance," Technical Report No. 95-12-01, Mechanical Engineering Department, University of Southern California.

T2 Fang, H., 1996, *Modeling and Analysis of One-Dimensional Complex Distributed Parameter Systems by the Distributed Transfer Function Method*, Ph. D. dissertation, University of Southern California.

## D. Advanced Degrees Received by Participating Scientific Personnel

| Name | Degree | Date |
|---|---|---|
| H. Fang | Ph. D. | August 1996 |
| D.-H. Park | Ph. D. | May 1997 (expected) |
| X. Wu | Ph. D. | August 1997 (expected) |

## A LIST OF REPORTABLE INVENTIONS

None